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Soft SUSY breaking contributions to proton decay

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Abstract: We show that in supersymmetric grand unified theories new effective D=4 and D=5 operators for proton decay are induced by soft SUSY-breaking terms, when heavy GUT gauge bosons are integrated out, in addition to the standard D=6 ones. As a result, the proton lifetime in gauge mediated channels can be enhanced or even suppressed depending on the size of the heavy Higgses soft terms.

Keywords: Supersymmetric gauge theory, GUT, Beyond Standard Model, Supersymmetry Breaking.

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Contents

1. **Introduction** 1

2. **Gauge mediated effective operators in softly broken SUSY GUT** 3

3. **Running and dressing** 5

4. **SU(5) example** 9

5. **Realistic models** 12

6. **Conclusions** 13

1. Introduction

One of the immediate consequences of Grand-Unified Theories (GUT) is baryon number non-conservation that can lead to proton decay [1, 2]. The heavy gauge bosons mediate the effective baryon-number violating four-fermion operators

$$O_{\rm gauge} \quad \sim \quad g_G^2 M_G^{-2}\,(\overline{q}\,u^c\,\overline{l}\,d^c)\,, \qquad g_G^2 M_G^{-2}\,(\overline{q}\,u^c\,\overline{q}\,e^c)\,, \tag{1.1}$$

where M_G and g_G are the Grand Unification scale and gauge coupling constant, $q = (u,d)_L$, $l = (\nu, e)_L$ are the left-handed quarks and leptons (weak isodoublets) and u^c, d^c, e^c are charge-conjugated fields of the right handed ones: u_R, d_R, e_R (weak isosinglets). In addition, other four-fermion operators of different structure are mediated by heavy colored triplet Higgses with mass $M_H \sim M_G$:

$$O_{\rm higgs} \quad \sim \quad g_Y^2 M_H^{-2}(q\,q\,q\,l)\,, \qquad g_Y^2 M_H^{-2}(u^c\,u^c\,d^c\,e^c)\,, \tag{1.2}$$

The latter operators are typically weaker because of smallness of the Yukawa couplings g_Y but in some models they can be dominant over the gauge mediated operators (1.1) [4].

The set (1.1), (1.2) represents all possible D=6 baryon number violating operators, independently of the details of grand unification [3]. The two kinds of operators have different chirality structures (LRLR for (1.1) and LLLL or RRRR for (1.2)) and they could in principle be distinguished via the polarization of final states [3]. Both kinds are suppressed by two powers of M_G.

In non supersymmetric models, when M_G is below 10^{15} GeV, processes following from (1.1) are already ruled out. Supersymmetry, in addition to making the unification more natural, raises its scale, setting the magnitude of these processes within the reach of near future experimental facilities.

Supersymmetry raises the unification scale to 10^{16} GeV and makes these operators hardly observable. On the other hand it introduces additional D=5 baryon number violating operators mediated by heavy colored higgsinos [5], as

$$O_{\rm higgsino} \quad \sim \quad g_Y^2 M_H^{-1} (\tilde{q}\, q\, \tilde{q}\, l)\,, \qquad g_Y^2 M_H^{-1} (\tilde{u}^c\, u^c\, \tilde{d}^c\, e^c)\,, \tag{1.3}$$

where tilded fields represent scalar superpartners. These are suppressed by a single power of the GUT scale and after dressing by gaugino exchange they give rise to operators of the form (1.2) with a cutoff scale $\sim (M_H m_S)^{1/2}$ where m_S is the SUSY-breaking scale [5]. They thus become generically dominant and on the verge of being in conflict with current experimental limits on these specific decay modes ($p \to K\nu$ etc.) [2]. However, their magnitude is very model dependent: essentially they exclude minimal versions GUTs and cause problems for models unless fine tuning is arranged. Several mechanisms have been devised to suppress them by playing with the structure of the heavy sector of the theories.[1]

Let us remark that gauge coupling unification does not strictly require supersymmetry of the theory. For instance the presence of fermionic partners of the gauge and higgs at TeV scale can adjust the running of the gauge coupling constants so that they unify at one point. Though scalars are not crucial for unification they are predicted by low scale SUSY; however, finding at LHC a SUSY-like spectrum would not mean that supersymmetry is discovered. Indeed, it would be extremely difficult to verify that the lagrangian has a supersymmetric structure, i.e. that the different coupling constants are related, like the quark-squark-gluino coupling constants that should be exactly the same as the strong gauge coupling constant.

One can imagine a *fake-SUSY* theory where only the sparticle *spectrum* is supersymmetric (i.e. every particle has its “superpartner”) while the the lagrangian is not. What would happen in such a theory? We argue that, even if the gauge coupling unification is achieved as perfectly as in a truly supersymmetric theory, it would lead to disastrous proton decay rate. The reason is the following: once such a theory contains scalars partners of quarks and leptons ($\tilde{q}$, $\tilde{l}$) it generically contains D=4 operators of the form

$$O_{\rm quartic} \quad \sim \quad (\tilde{q}^*\, \tilde{u}^c\, \tilde{l}^*\, \tilde{d}^c)\,, \qquad (\tilde{q}^*\, u^c\, \tilde{q}^*\, e^c)\,, \tag{1.4}$$

which in a GUT context can not be excluded by any symmetry reason. Notice that even if they are not present in the bare lagrangian they emerge radiatively by loops of GUT gauge bosons. The dressing by gauginos transforms these D=4 into D=6 ones on the form (1.1) that directly cause the proton to decay at a dangerous high rate, being suppressed only by two powers of the fake superpartners mass scale that is of order TeV.

Complete supersymmetry instead provides an automatic protection from these D=4 operators: they in fact correspond to the D-terms relative to the broken gauge generators, and they have to vanish if SUSY is unbroken. What happens is that the existing D-term

[1] There are several ideas how dimension-5 operators can be suppressed. In particular this can be due to special arrangements in the heavy higgs sector [6], because of symmetry properties of the Yukawa sector [7]. In SO(10) models, LLLL operators can be naturally suppressed by the choice of the SO(10) breaking VEVs while the less dangerous RRRR ones are left allowed. With further model building also these latter can be eliminated [8]. Finally, in supersymmetry there are also D=4 B and L violating operators that can be forbidden by exact R-parity [9].

involving light fields $g^2|\phi^*\phi|^2$ is cancelled by two other diagrams: one with the exchange of the (broken) gauge field and one with the exchange of the heavy longitudinal part of the higgs field that breaks the gauge group. For this cancellation to hold it is crucial that the coupling g is exactly the same in the three graphs, i.e. that SUSY is exact. The shortest proof of this fact can be given in the superfield formalism, where the only supersymmetric D-term involving four light superfields is $[\Phi^\dagger\Phi\Phi^\dagger\Phi]_D$. This operator does not contain a four scalar contact interaction, that therefore has to vanish.

However, since supersymmetry has to be broken, one expects that this protection mechanism works only partially and that the susy-breaking terms will turn on such operators. As a result they may significantly affect the proton decay. Indeed, it was noted in [10] that SUSY-breaking induces the D=4 scalar operators (1.4). However, surprisingly enough a complete analysis of the soft-susy breaking effects on proton decay has not been performed.[2]

In this work we study the effect of the soft terms on the low energy effective theory produced after the heavy gauge superfields are integrated out at the GUT scale. We show that the D=6 operators are always accompanied by new operators of D=5 and D=4, turned on by the presence of the soft terms. Next, we compute the renormalization of these operators from the GUT scale to the SUSY breaking scale; we adopt the techniques illustrated in [15] that simplify considerably the task. We then dress the new D=5 and D=4 operators at the SUSY breaking scale, transforming them in the form (1.1) and estimate when they can be relevant. As an example, we discuss the SUSY SU(5) model and show that their contribution can be important and could bring the proton decay rate in specific channels to be experimentally accessible.

2. Gauge mediated effective operators in softly broken SUSY GUT

The gauge mediated effective operators are efficiently described in the superfield formalism with soft breaking terms inserted as spurions. If we arrange the chiral superfields of irreducible representations in a column vector $\Phi = \{\Phi_I\}$, the full lagrangian is, in compact notation:

$$\mathcal{L} = \int \mathrm{d}^4\theta \left[\Phi^\dagger X \mathrm{e}^{2gV}\Phi\right] + \int \mathrm{d}^2\theta \left[W(\Phi) + W_\alpha W^\alpha Y\right] + h.c.\,. \tag{2.1}$$

Here the gaugino masses enter via $Y = (1 + m_{\tilde{g}}\theta^2)$, and the soft D-terms may be parametrized by the matrix $X = (1 + \Gamma\theta^2 + \Gamma^\dagger\bar{\theta}^2 + Z\theta^2\bar{\theta}^2)$ via the matrices $\{\Gamma_{IJ}\}$ of order m_S and $\{Z_{IJ}\}$ of order m_S^2. One can however perform a field redefinition to set $\Gamma_{IJ} = 0$. For simplicity we will also consider only universal soft terms, taking Z_{IJ} diagonal. Therefore we have:

$$X_{IJ} = X_I\,\delta_{IJ} = (1 - m_I^2\theta^2\bar{\theta}^2)\,\delta_{IJ}\,. \tag{2.2}$$

The superpotential $W(\Phi)$ includes the soft F-terms via spurion fields and may be parametrized similarly (see e.g. (4.1)) but its explicit form is not directly relevant for this section.

[2] In [11] it is described a classification of all the D=4, 5, 6 operators relevant for proton decay, while in [12] the effect of soft terms in a SUGRA scenario was studied, but only for the analytic D=5, 4 operators.

At the scale of gauge symmetry breaking one can decompose Φ_I in $(\Phi_H,\ \Phi_A,\ \phi_i)$, respectively heavy superfields, light goldstone superfields, and light non-goldstone superfields. The decoupling of heavy superfields Φ_H (e.g. colored higgses) leads to dimension-6 and analytic dimension-5 effective operators that may violate baryon number. The decoupling of heavy gauge fields and goldstones in turn leads to the D-term effective operators of dimension 6 [5]. All these operators are affected by the soft susy breaking terms.

To find the effect on the gauge mediated dimension-6 operators, it is convenient to adopt the so called super-unitary gauge [14], where the goldstone superfields Φ_A are gauged away inside the broken massive gauge superfields, denoted as V_A. To integrate out V_A one expands the gauge exponential in (2.1) to the quadratic order

$$\begin{aligned} \mathcal{L}_{(2)} &= \int \mathrm{d}^4\theta \Big[\Phi^\dagger X \Phi + 2 J_A V_A + K_{AB} V_A V_B + \cdots \Big] \\ J_A &= g\, \Phi^\dagger X T_A \Phi\,, \\ K_{AB} &= 2g^2 \Phi^\dagger X T_A T_B \Phi \end{aligned} \tag{2.3}$$

and then one notes that in the unitary gauge

$$J_A = g\, \phi_i^\dagger T_A \phi_i\, X_i \qquad K_{AB} = 2g^2 \langle\Phi_H\rangle^\dagger T_A T_B \langle\Phi_H\rangle\, X_H\,.$$

As expected a VEV of the heavy fields give a squared-mass matrix K_{AB} to the broken gauge fields.[3] In a suitable basis of broken generators this matrix is diagonal, $K_{AB} = K_A \delta_{AB}$. We can note already at this stage that the heavy gauge boson mass matrix contains SUSY-breaking factors, such as the X's.

The result after integrating out the broken gauge fields V_A is then:

$$\begin{aligned} &\int \mathrm{d}^4\theta\, J_A K^{-1}_{AB} J_B = \\ &= \int \mathrm{d}^4\theta\, \frac{g^2}{2g^2 \langle\Phi_H\rangle^\dagger T_A T_A \langle\Phi_H\rangle}\, \frac{X_i X_j}{X_H} \Big(\phi_i^\dagger T_A \phi_i\Big) \Big(\phi_j^\dagger T_A \phi_j\Big)\,, \end{aligned} \tag{2.4}$$

where summation on all indices is understood. Note that in the integration we have ignored sub-leading terms like the gauge kinetic term and the gaugino masses for the V_A gauge fields. In fact at they both give subleading effects in (2.4).

Considering that $\langle\Phi_H\rangle \sim M_G$, we recognize in the first factor the standard coupling constant of the dimension-6 operators $\sim 1/M_G^2$. The supersymmetry breaking however has propagated in this operator, and indeed the second factor involves the soft SUSY breaking D-terms X that are carried along in the decoupling process. Moreover, due to the soft SUSY breaking in the superpotential, also $\langle\Phi_H\rangle$ has in general a non-vanishing F-term, $\langle\Phi_H\rangle = v_H(1+f_H\theta^2)$, that induces an other supersymmetry breaking in the effective gauge bosons mass.

[3] In the presence of non-universal soft terms J_A has an additional piece $\langle\Phi_H\rangle^\dagger T_A X_{Hj}\phi_j$, that gives new soft masses to ϕ_j. Also K_{AB} is modified in a similar fashion, see [13]. However the present analysis is not affected substantially.

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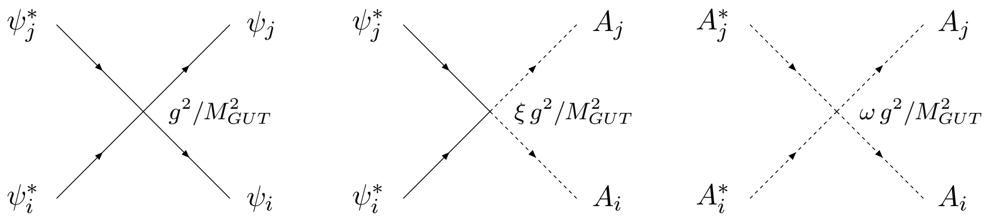

Figure 1: New operators generated by soft SUSY breaking terms in the heavy gauge exchange.

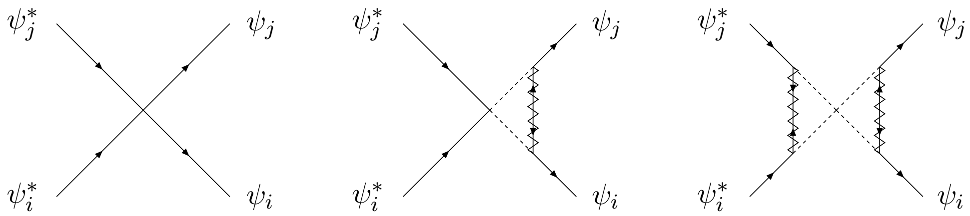

Figure 2: four-fermion operators after dressing via gluino exchange.

The overall result with SUSY-breaking terms can be conveniently rewritten as:

$$\sum_{ij,A} \int \lambda_6 \Big(1 + \xi\theta^2 + \xi^\dagger\bar\theta^2 + \omega_{ij}\theta^2\bar\theta^2\Big) \Big(\phi_i^\dagger T_A \phi_i\Big) \Big(\phi_j^\dagger T_A \phi_j\Big)\, \mathrm{d}^4\theta\,. \tag{2.5}$$

where $\lambda_6 = g^2/M_A^2$ is the supersymmetric four-fermion coupling, the heavy gauge-bosons masses are given by $M_A^2 = \sum_H 2g^2(v_H^\dagger T_A T_A v_H)$, and the SUSY-breaking coefficients are:

$$\xi = -f_H\,, \qquad \omega_{ij} = -m_i^2 - m_j^2 + m_H^2 + |f_H|^2\,. \tag{2.6}$$

For simplicity in this last expression we have assumed the breaking by a single VEV.[4]

In terms of field components the effective operator (2.5) contains the three operators shown in figure 1: the standard dimension-6 four-fermion operator $\psi^*\psi\psi^*\psi$ with coupling $\sim 1/M_G^2$; then a new dimension-5 operator of the form $A^*\psi A^*\psi + h.c.$ with coupling $\sim m_S/M_G^2$ coming from the terms with θ^2 and $\bar\theta^2$, and finally a new dimension-4 operator of the form AA^*AA^*, with coupling $\sim m_S^2/M_G^2$.

The new dimension-5 and dimension-4 operators can be dressed by gaugino exchange at the SUSY-breaking scale (see figure 2) and transformed in effective dimension-6 four-fermion operators, as it happens for dimension-5 analytic operators. Each dressing loop brings a factor $\sim 1/m_S$, so that the effective strength of all these operators is the same, $1/M_G^2$. The actual relative strength will depend on the coupling constants involved in the dressing and on the ratio of the effective soft breaking parameters ξ, ω to the gaugino and/or sfermion masses.

3. Running and dressing

To calculate the effect of the three operators in (2.5) one has to run them from the decoupling (GUT) scale down to the SUSY breaking scale and dress them to get the dimension-6

[4] With more VEVs, in the first formula f_H should be replaced by its *average* $(\sum_H M^2_{A\,(H)} f_H)/(\sum_H M^2_{A\,(H)})$, where $M^2_{A\,(H)} = 2g^2 v_H^\dagger T_A T_A v_H$. Similarly in the second formula for m_H^2 and $|f_H|^2$.

effective operators. The running below the SUSY scale is non-supersymmetric and was analyzed in [16].

Renormalization mixes the supersymmetric and the non-supersymmetric effective operators via the soft susy breaking parameters of the theory, mainly the gaugino masses. The detailed computation is rather complicated due to the large number of diagrams involved.

Instead of attacking the problem by brute force, we employ the elegant techniques devised in [15] to analyze the soft terms renormalization. Starting from the anomalous dimensions of the supersymmetric operators, we find the renormalization in the softly broken theory by promoting the couplings to full superfields built with the soft terms.

We start from the definition of the renormalization of the supersymmetric coupling in (2.5):

$$\frac{\lambda_6^B}{\lambda_6} = Z_6(\alpha_3,\alpha_2,\alpha_1)\,, \qquad Z_6 = \prod_{i=1,2,3}\left(\frac{\alpha_i^B}{\alpha_i}\right)^{\frac{-\gamma_6^{(i)}}{b^{(i)}}} \tag{3.1}$$

where $b^{(i)}$ is the beta-function coefficient for each gauge group and $\gamma_6^{(i)}$ is the corresponding supersymmetric contribution to the anomalous dimension of λ_6.[5] These anomalous dimensions were calculated in [17]: $\gamma_6^{(3)} = -4/3$, $\gamma_6^{(2)} = -3/2$, $\gamma_6^{(1)} \simeq -23/30$.

The key step, to renormalize the full operator $\lambda_6(1+\xi\theta^2+\xi^\dagger\bar\theta^2+\omega\bar\theta^2\bar\theta^2)$ in the presence of soft terms, is to promote each gauge coupling α to $\tilde\alpha = \alpha(1+m_{\tilde g}\theta^2+m_{\tilde g}^*\bar\theta^2+2|m_{\tilde g}|^2\theta^2\bar\theta^2)$:

$$\frac{\lambda_6^B(1+\xi^B\theta^2+\xi^{\dagger B}+\omega^B\bar\theta^2\bar\theta^2)}{\lambda_6(1+\xi\theta^2+\xi^\dagger\bar\theta^2+\omega\bar\theta^2\bar\theta^2)} = Z_6(\tilde\alpha_3,\tilde\alpha_2,\tilde\alpha_1)\,. \tag{3.2}$$

Expanding then Z_6 in grassmann variables we find how the operators of different dimension mix under renormalization:

$$4\pi\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\lambda_6\\ \lambda_6\xi\\ \lambda_6\xi^\dagger\\ \lambda_6\omega\end{pmatrix} = \gamma_6\alpha\begin{pmatrix}1 & 0 & 0 & 0\\ m_{\tilde g} & 1 & 0 & 0\\ m_{\tilde g}^* & 0 & 1 & 0\\ 2|m_{\tilde g}|^2 & m_{\tilde g}^* & m_{\tilde g} & 1\end{pmatrix}\begin{pmatrix}\lambda_6\\ \lambda_6\xi\\ \lambda_6\xi^\dagger\\ \lambda_6\omega\end{pmatrix}, \tag{3.3}$$

where $t=\ln(\mu^2)$ and a summation on the different gauge groups is implicit in the r.h.s.. The gaugino masses $m_{\tilde g}$ and gauge couplings α follow the equations $\dot m_{\tilde g}/m_{\tilde g} = \dot\alpha/\alpha = b\alpha/4\pi$.

The equations (3.3) are solved in terms of the evolution of the gauge coupling constants α from the GUT to the SUSY scale by using the auxiliary functions

$$R = \frac{\alpha(S)}{\alpha(G)}\,, \qquad R_1 = \frac{\gamma_6}{b}(R-1)\,, \qquad R_2 = \frac{\gamma_6}{b}(R^2-1)\,.$$

[5]In the one-loop approximation only the renormalization due to gauge loops needs to be taken into account, and in the operators involving the first generation the contribution of large top Yukawa is suppressed by mixings. In higher loop orders one should also include other effects, for example the threshold corrections due to insertions of more than one λ_6.

JHEP10(2006)030

The result is:

$$\begin{aligned}
\lambda_6(S) &= R^{-\frac{\gamma_6}{b}}\,\lambda_6(G) \\
\xi(S) &= \xi(G) + R_1\, m_{\tilde g}(G) \\
\omega(S) &= \omega(G) + 2R_1\ \mathrm{Re}[\xi(G)\, m^*_{\tilde g}(G)] + \left(R_2 + R_1^2\right) m^2_{\tilde g}(G)\,.
\end{aligned}$$

For example, the largest effect comes from SU(3) color, for which $\gamma_6^{(3)} = -4/3$, $b^{(3)} = -3$, $\alpha_3(M_Z) = 0.119$ and $\alpha_3(G) = 1/23$:

$$\lambda_6(S) \simeq 1.55\,\lambda_6(G)\,, \tag{3.4}$$

$$\xi(S) \simeq \xi(G) + 0.74\, m_{\tilde g 3}(G)\,, \tag{3.5}$$

$$\omega(S) \simeq \omega(G) + 1.48\,\xi(G)\, m_{\tilde g 3}(G) + 3.26\, m^2_{\tilde g 3}(G)\,, \tag{3.6}$$

where for simplicity we assumed ξ and $m_{\tilde g}$ real.

The effective strength of the dimension 6, 5 and 4 operators can be compared after dressing with exchange of some gaugino. As shown in figure 2, it is clear that the chiral structure of the D=5 operators requires a Majorana mass to perform a chirality flip, and for low momentum processes as proton decay this is true also for the D=4 operators. The D=5 operators can be dressed by gluino exchange; the D=4 operators on the other hand can only involve one gluino exchange while the other loop is necessarily formed via W-ino (or B-ino exchange).

We can estimate the strength of the two new operators by defining the corresponding effective four-fermion couplings at the SUSY scale:

$$\begin{aligned}
\lambda_5 &= \lambda_6\, 2\,\xi\, \frac{\alpha_3}{4\pi}\, L(m_{\tilde x}, m_{\tilde y}, m_{\tilde g 3}) \\
\lambda_4 &= \lambda_6\ \omega\, \frac{\alpha_3}{4\pi}\, L(m_{\tilde x}, m_{\tilde y}, m_{\tilde g 3})\, \frac{\alpha_2}{4\pi}\, L(m_{\tilde x'}, m_{\tilde y'}, m_{\tilde g 2})
\end{aligned}$$

where $m_{\tilde x, \tilde y}$ are the sfermion masses entering the dressing loop(s) and α_2, α_3 the gauge coupling constant involved. All quantities are evaluated at the SUSY scale. L is the loop integral:

$$\begin{aligned}
L(m_1, m_2, m_3) &= m_3 \frac{m_1^2 m_2^2 \log\frac{m_1^2}{m_2^2} + m_2^2 m_3^2 \log\frac{m_2^2}{m_3^2} + m_1^2 m_3^2 \log\frac{m_3^2}{m_1^2}}{\left(m_1^2 - m_2^2\right)\left(m_1^2 - m_3^2\right)\left(m_2^2 - m_3^2\right)} \\
&= \frac{1}{m_3}\, \frac{\frac{m^2}{m_3^2} - 1 - \log\frac{m^2}{m_3^2}}{\left(\frac{m^2}{m_3^2} - 1\right)^2} \qquad \text{for } m_1 = m_2 = m
\end{aligned} \tag{3.7}$$

The loop integral is plotted in figure 3, where all the masses are measured in TeV. From there we see that L may be of order $1\,\mathrm{TeV}^{-1}$ (with a maximum of $L \sim 5$–$6\,\mathrm{TeV}^{-1}$) for small sfermion masses $\sim 100\,\mathrm{GeV}$. On the other hand the gaugino mass may be raised up to 1–2 TeV before starting to suppress the loop.

JHEP10(2006)030

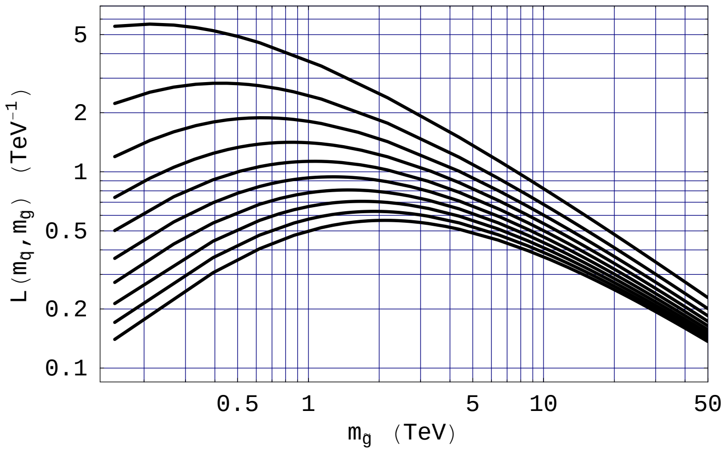

Figure 3: Loop function L in TeV^{-1}, with $m_{\tilde{q}}$ ranging from 0.1 (upper) to 1 TeV (lower) in steps of 100 GeV.

The present limits [27] allow squark masses as low as 100 GeV when the gluino mass is $\gtrsim 500\,\mathrm{GeV}$. In this parameter region, we find that L is almost maximal, $L \sim 5$, so for the numerical estimates we will stick to this choice. A different choice can be easily considered by extracting the relevant loop factor from eq. (3.7) or directly from figure 3.

With this choice the strengths of the new operators relative to the D=6 one are:

$$\lambda_5/\lambda_6 \simeq \frac{\alpha_3}{4\pi}\, 10\, \frac{\xi}{\mathrm{TeV}} \simeq \frac{\xi}{10\,\mathrm{TeV}}$$
$$\lambda_4/\lambda_6 \simeq \frac{\alpha_3}{4\pi}\frac{\alpha_2}{4\pi}\, 25\, \frac{\omega}{\mathrm{TeV}} \simeq \frac{\omega}{(30\,\mathrm{TeV})^2}\,. \tag{3.8}$$

As a result, the effect of D=5 and D=4 operators may be comparable (or larger) than that of the standard D=6 operators. However for this to happen the effective susy-breaking terms ξ and ω should be larger than the soft masses. One needs for example $\xi \simeq 10\,\mathrm{TeV}$, a factor of 20 or 100 larger than the gaugino or sfermion masses.

One should also ask whether these large values might be generated in the evolution of 13 orders of magnitude from the GUT down to the SUSY scale, by mixing with other soft parameters, namely the gaugino masses. However from the running (3.6) we see that in the regime of $\xi, \omega \gg m_{\tilde{g}3}$ the gaugino gives a small contribution that does not modify the estimate (3.8).

Is it then plausible for ξ or ω at GUT scale to be so larger than other soft susy breaking parameters in the theory? We argue that this is possible without spoiling the framework of low energy supersymmetry. The reason is as follows: from the expression of ξ and ω, eq. (2.6), we see that they are induced, in addition to the soft masses of fermion fields, by m_H^2 and f_H, the soft SUSY-breaking parameters in the heavy higgs sector. One can not play much with the soft masses of the heavy fields m_H^2, since these are constrained because they usually mix with the MSSM higgses soft masses in the renormalization from the Planck to the GUT scale. On the other hand the F-terms f_H are less constrained, since they do not directly enter in the running and one should not assume them to be small. This can be seen in the minimal SU(5) model as we illustrate in the following section.

JHEP10(2006)030

4. SU(5) example

In minimal supersymmetric SU(5) [18] the GUT breaking is due to the VEV of an adjoint superfield $\Sigma \in \mathbf{24}$. The superpotential for Σ includes the soft terms A_Σ, B_Σ as follows:

$$W(\Sigma) = M_\Sigma \,\mathrm{tr}\, \Sigma^2(1 - B_\Sigma\theta^2) + \frac{1}{6}\lambda_\Sigma \,\mathrm{tr}\, \Sigma^3(1 - A_\Sigma\theta^2)\,. \tag{4.1}$$

The effect of A_Σ and B_Σ is to give an F-term to $\langle\Sigma\rangle$ and to shift its magnitude by a small amount:

$$\langle\Sigma\rangle = v_\Sigma\left[1 + (A_\Sigma - B_\Sigma)\theta^2\right]\lambda_Y\,, \qquad v_\Sigma = 8\sqrt{15}\frac{M_\Sigma}{\lambda_\Sigma}\left[1 + \frac{A_\Sigma - B_\Sigma}{2M_\Sigma}\right] \simeq 8\sqrt{15}\frac{M_\Sigma}{\lambda_\Sigma}\,, \tag{4.2}$$

where $\lambda_Y = \mathrm{diag}(2,2,2,-3,-3)/\sqrt{60}$.

The fermion multiplets in SU(5) are **10** and $\bar{\mathbf{5}}$, and proton decay can proceed via the four field operators involving the combinations $\mathbf{10}$-$\mathbf{10}$-$\bar{\mathbf{5}}$-$\bar{\mathbf{5}}$ or **10-10-10-10**. At GUT scale the standard D=6 operator has coupling constant

$$\lambda_6(G) = g_5^2/M_A^2\,, \qquad \text{with } M_A^2 = 5\, g_5^2 v_\Sigma^2/12\,, \tag{4.3}$$

where g_5 is the SU(5) gauge coupling. Using eq. (2.6) we find the coefficients of the new operators:

$$\begin{aligned}\xi = B_\Sigma - A_\Sigma\,, \qquad \omega_{\mathbf{10}\,\bar{\mathbf{5}}} &= -m_{\mathbf{10}}^2 - m_{\bar{\mathbf{5}}}^2 + m_\Sigma^2 + |B_\Sigma - A_\Sigma|^2 \\ \omega_{\mathbf{10}\,\mathbf{10}} &= -2m_{\mathbf{10}}^2 + m_\Sigma^2 + |B_\Sigma - A_\Sigma|^2\,,\end{aligned} \tag{4.4}$$

where $m_{\bar{\mathbf{5}}}^2$, $m_{\mathbf{10}}^2$ and m_Σ^2 are the soft masses of the $\bar{\mathbf{5}}$, **10** fermion multiplets and of Σ itself.

In the previous section we found that the new operators for proton decay are relevant when the squark masses are small while ξ or ω are larger, $\sim 10\,\mathrm{TeV}$. From (4.4) we see that this may be realized when the soft mass m_Σ^2 or the analytic soft terms $B_\Sigma - A_\Sigma$ are large. A large m_Σ^2 is not appealing, since m_Σ enters the RG running of the Higgs soft masses and would induce large values for these, spoiling the picture of electroweak breaking. The same holds for A_Σ, since it also enters in the running of m_Σ^2 and other soft parameters (see e.g. [20]), and a large A_Σ would indirectly cause color breaking minima. On the other hand we note that B_Σ does not enter the evolution of other quantities and may be sensibly large, without driving all the other soft parameters to large values as well.

The fact that soft B-terms do not enter in any beta function is a general statement valid in the MS scheme, that follows from SUSY and the fact that in this scheme no spurious scales are introduced. It turns out that the soft B-terms like B_Σ, being of dimension one, never enter any RG equation, at all loops. Specifically, this can be verified in RG equations of soft masses, for A-terms and for B-terms themselves; for example we have, for the one-loop running of A_Σ and B_Σ from Planck to GUT scale:[6]

$$16\pi^2\frac{\mathrm{d}}{\mathrm{d}t}A_\Sigma = \frac{63}{20}A_\Sigma\lambda_\Sigma^2 + 3A_H\lambda_H^2 - 30\, g_5^2\, m_{\tilde{g}_5} \tag{4.5}$$

[6]The constants λ_H, A_H are defined below, (4.8). To compare with evolution of other quantities see e.g. [20].

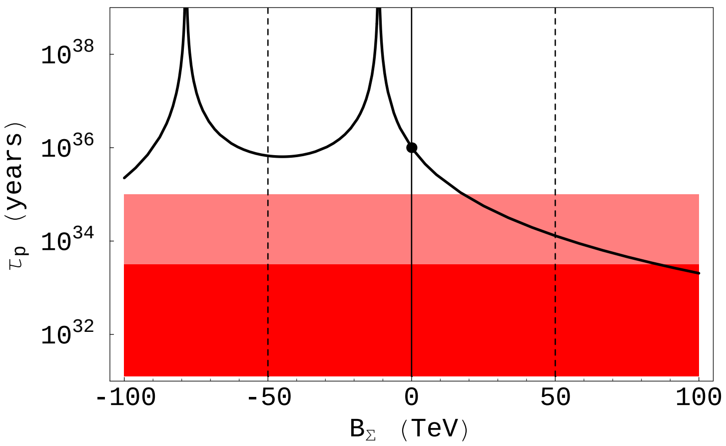

Figure 4: Effect of dimension 4 and 5 on gauge mediated proton decay as a function of the soft SUSY-breaking terms in the heavy sector. The dot represents our reference value for the supersymmetric gauge mediated proton lifetime, taken to be 10^{36} y. The shaded regions show current and 10-years expected limits on the $p \to \pi^0 e^+$ partial lifetime. Dashed lines mark the limit where B_Σ start to affect the higgs soft masses and other soft parameters.

$$16\pi^2 \frac{\mathrm{d}}{\mathrm{d}t} B_\Sigma = \frac{21}{10} A_\Sigma \lambda_\Sigma^2 + 2 A_H \lambda_H^2 - 20\, g_5^2\, m_{\tilde{g}_5}\,. \tag{4.6}$$

From these equations one can also see that if a large B_Σ is generated at the Planck scale, it will not be substantially affected running down to the GUT scale.

Of course one can not hope to raise one soft parameter without consequences. Even if B_Σ does not appear in RG equations, going from the MS to a physical scheme, it will enter in finite corrections. In particular a large B_Σ will generate corrections to soft quantities [19, 20], raising them as if actually SUSY were broken at the B_Σ scale. As we discuss below, this effect is relevant only when B_Σ exceeds ~ 50 TeV, with some model dependence. Below this limit the only physical effect of a large B_Σ is in the soft ξ and ω coefficients, where it can directly dominate in the D=5 and D=4 operators and thus enhance or even suppress the proton decay rate.

To give a concrete estimate in the SU(5) example, we assume large B_Σ and use the soft coefficients $\xi \simeq B_\Sigma$ and $\omega \simeq |B_\Sigma|^2$ in eq. (3.8), to find how the proton lifetime for a gauge-mediated channel like $p \to \pi^0 e^+$ is modified:

$$\tau_{SOFT}^{-1} \simeq \tau_{SUSY}^{-1} \left(1 + \frac{B_\Sigma}{10\,\mathrm{TeV}} + 0.1 \left| \frac{B_\Sigma}{10\,\mathrm{TeV}} \right|^2 \right)^2 . \tag{4.7}$$

An explicit plot of the effect of large B_Σ is shown in figure 4, where we assume a reference value of 10^{36} y for the proton partial lifetime.[7] We see that the effect can be rather evident: for example for negative B_Σ the proton decay can be made absolutely unobservable, while for B_Σ positive one can enter in the region of sensitivity of the next ten years water-cerenkov detectors [21]. We conclude that large soft SUSY-breaking terms

[7] This reference value corresponds to a Grand Unification scale of $2 \cdot 10^{16}$ GeV, and we remind that τ_{SUSY}^{-1} scales as the fourth power of M_G, which is model dependent.

JHEP10(2006)030

for the heavy fields may significantly affect the proton decay rate even in the gauge mediated channels.

Let us now address the fine tuning problems related to the stability of the hierarchy in presence of a large B_Σ. This point is related to the problem of doublet-triplet splitting, as can be seen in the SU(5) example.

The superpotential involves also the higgses H, $\overline{H}$ (transforming in the $\mathbf{5}$, $\bar{\mathbf{5}}$)

$$W(\Sigma, H, \bar{H}) = W(\Sigma) + M_H(1 - B_H\theta^2)\bar{H}H + \lambda_H(1 - A_H\theta^2)\bar{H}\Sigma H\,. \tag{4.8}$$

where $W(\Sigma)$ is given in (4.1). After the SU(5) breaking H, $\overline{H}$ leave the light MSSM higgs doublets H_u, H_d, with their soft masses m_u^2, m_d^2, and we get also the effective μ and B_μ terms

$$\mu(1 - B_\mu\theta^2)H_uH_d \tag{4.9}$$

where

$$\mu = M_H - \frac{3}{\sqrt{60}}\lambda_H v_\Sigma\,, \tag{4.10}$$

$$\mu B_\mu = \frac{3}{\sqrt{60}}\lambda_H v_\Sigma(A_\Sigma - B_\Sigma - A_H + B_H) + O(B_\Sigma - A_\Sigma)^2\,. \tag{4.11}$$

Therefore two fine-tuning conditions are needed to achieve the electroweak symmetry breaking at the correct scale, one for μ and another for B_μ. In fact the mass matrix of the higgs scalars is:

$$\begin{pmatrix} \mu^2 + m_u^2 & \mu B_\mu \\ \mu B_\mu & \mu^2 + m_d^2 \end{pmatrix} \tag{4.12}$$

and all entries should be of the order of the electroweak scale.

The second fine tuning (4.11) can be avoided by assuming universality of A and B terms separately, $A_\Sigma = A_H$ and $B_\Sigma = B_H$, as noticed in [23], so that $(A_\Sigma - B_\Sigma - A_H + B_H) = 0$ and B_μ is of the order of soft susy-breaking scale. In the case of large $B_\Sigma \sim 10\,\text{TeV}$ however one still gets a B_μ term that is too large, therefore the right pattern of electroweak breaking can be obtained only by tuning the two independent parameters, the supersymmetric μ and soft B_μ.

Of course, this minimal SU(5) model is not realistic, and one should not be surprised to find that fine tunings are required. In the next section we describe how in specific models fine tunings can be avoided and one can have large B-terms without spoiling the hierarchy.

Before moving to more realistic models, we point out that generically there are also finite corrections induced by B-terms. For example in SU(5) B_Σ induces a shift of the analytic soft term for the higgses B_μ, of their soft masses and also of the gaugino masses. These corrections are loop suppressed:

$$\delta B_\mu \sim \frac{\lambda_H^2}{(4\pi)^2}B_\Sigma\,, \qquad \delta m_{\tilde{g}} \sim \frac{g_5^2}{(4\pi)^2}B_\Sigma\,, \tag{4.13}$$

with some model dependent numerical factors [22]. Since $\lambda_H \simeq g_5 \simeq 0.7$, the loop suppression factor is $\sim 1/100$, and we conclude that these corrections can be ignored as far as $B_\Sigma < 50\,\text{TeV}$. Beyond this limit the gaugino mass and the higgs mass terms would need some fine tuning, to avoid breaking SUSY at a high effective scale or having an unacceptably large higgs mass.

5. Realistic models

In minimal SU(5) model the problem of doublet-triplet splitting has only a technical solution: fine tuning of μ, eq. (4.10) that is stable against radiative corrections; the situation is then worsened by the need of another fine tuning in the soft terms, eq. (4.11).

Moreover, to achieve the right electroweak scale of order 100 GeV with a large $B_\mu \sim 10\,\text{TeV}$ would require a fine tuning with the μ term, while there is no apriori correlation between these two parameters.

This problem gets another twist in realistic models in which the doublet-triplet splitting problem is solved without fine tuning. In particular, in SU(5) this can be done via the "Missing Doublet Mechanism" (MDM) [24], and in SO(10) via the "Missing VEV Mechanism" (MDM) [25], while in SU(6) via the "Goldstones instead of Fine Tuning" (GIFT) Mechanism [26].

In particular, in all these models the soft parameters like B_Σ or A_Σ for the heavy GUT breaking superfields can be taken much larger than that of matter superfields, without creating additional fine-tuning problems. Let us briefly describe them here.

In SU(5), the missing doublet model [24] contains the Higgs superfields in representations $\Phi \sim 75$, $H \sim 5$, $\bar{H} \sim \bar{5}$, $\Psi \sim 50$, $\bar{\Psi} \sim \overline{50}$, with the following superpotential terms:

$$W = M\Phi^2 + \lambda\Phi^3 + M_1\Psi\bar{\Psi} + \lambda_1 H\Phi\bar{\Psi} + \lambda_2\bar{H}\Phi\Psi + \mu H\bar{H} \tag{5.1}$$

with M and M_1 being the mass parameters order M_G and λ's being the order 1 coupling constants. SU(5) is broken to SU(3)×SU(2)×U(1) by the VEV of Φ which also generates the mixing between the color triplet fragments in the Higgs 5- and 50-plets, whereas there are no doublets in the 50-plets. In this way, all color triplets are heavy, with mass of order M_G, while the doublets in $H, \bar{H}$ remain light, with mass given by the μ-term. Obviously, in this theory the soft parameter B_Φ can be taken large without inducing a large B_μ (still inside the limits set by the induced finite corrections like (4.13)).

For SO(10), in the missing VEV model [25], the philosophy is similar: the Higgs doublets remain massless because the GUT-breaking fields have zero VEV along the direction that would give them a mass, whereas it couples to the triplets with non-zero VEV. Therefore also in this case the protection of the doublet sector is due to group theoretical reasons, therefore large soft terms $\sim 10\,\text{TeV}$ in the heavy sector will not influence μ and B_μ and the electroweak scale will not be destabilized.

In the SU(6) model [26], the SU(6) gauge symmetry is broken by two sets of superfields: one contains an adjoint representation $\Sigma \sim 35$, that leads to the breaking channel SU(6)→SU(4)× SU(2)× U(1), and the other contains two fundamental representations $H \sim 6$ and $\bar{H} \sim \bar{6}$ that break SU(6)→SU(5). As a result the two channels together break the SU(6) gauge symmetry down to SU(3)×SU(2)×U(1). Also, one assumes that the Higgs superpotential does not contain the mixed term $H\Sigma\bar{H}$, so that it has the form $W = W(\Sigma) + W(H,\bar{H})$, where

$$W(\Sigma) = M\Sigma^2 + \lambda\Sigma^3\,, \qquad W(H,\bar{H}) = Y(H\bar{H} - V^2)\,. \tag{5.2}$$

As a result there is an accidental global symmetry $\mathrm{SU}(6)_\Sigma\times\mathrm{SU}(6)_H$, which independently transform Σ and $H,\bar{H}$ superfields. Then, in the limit of unbroken supersymmetry the MSSM Higgs doublet H_u, H_d appear as massless goldstone superfields built up as a combination of doublet fragments from Σ and H, $\bar{H}$, that remain uneaten by the gauge bosons. Therefore in this limit μ vanishes exactly.

Supersymmetry breaking terms like A_Σ, B_Σ shift the VEVs and also give F-terms to them, therefore generating B_μ term for the MSSM Higgses. However, since these terms also respect the global symmetry $\mathrm{SU}(6)_\Sigma\times\mathrm{SU}(6)_H$, the mass matrix of the Higgses (4.12) is degenerate and so one Higgs scalar (combination of the scalar components of H_u and H_d) still remains massless. Thus, even with arbitrary B_Σ that give $\mu \sim B_\mu \sim B_\Sigma$, there is an automatic relation between μ and B_μ terms that guarantees that the determinant of (4.12) vanishes.

This degeneracy is removed only by radiative corrections due to Yukawa terms that do not respect the global symmetry, and the resulting Higgs mass will be of the order of μ and B_μ, given by the mismatch in their renormalization. Therefore, in the case of large $B_\Sigma \sim 10\,\mathrm{TeV}$ we are still left with a "little" hierarchy problem of the electroweak scale stability against 10 TeV. However by enlarging the gauge symmetry this issue can be avoided. In fact one can have that 10 TeV is only an intermediate scale where an extra global symmetry guarantees the protection of the electroweak scale, the so called super-little-higgs mechanism [28].

6. Conclusions

In this paper we have studied the effects of soft SUSY-breaking terms on proton decay in SUSY GUT theories. While the dominant effect in SUSY GUT comes from D=5 higgs-mediated operators, these are very model dependent and may be suppressed by specific constructions. Here we have focused on gauge mediated effective operators, that are usually unavoidable.

We have shown how soft terms enter into the gauge-mediated effective operators for proton decay: while the supersymmetric operators are of dimension 6, SUSY breaking always induces new operators of dimension 5 and 4.

We computed their renormalization from the GUT to the SUSY scale, that amounts to a small mixing of the D=6, 5, 4 operators through the gaugino masses.

The new operators are dressed via gaugino exchange and transformed into D=6 four-fermion operators, and have the same suppression factor M_G^{-2} of the standard D=6 operators. They however have numeric coefficients that depend on the ratio of soft-breaking parameters in the heavy and light sectors.

When all the soft breaking parameters are of the same order, the dressing loop factors are small enough to suppress these new operators. However, we note that the B-terms

JHEP10(2006)030

in the heavy-Higgs sector may be substantially higher than the standard soft masses, and they do not mix with soft masses under renormalization. Finite corrections are present which are irrelevant when the heavy B-terms are smaller than $\sim 50\,\text{TeV}$.

The heavy higgses soft-terms then enter the GUT breaking process and lead to observable effects on D=6 proton decay. B-terms as low as $10\,\text{TeV}$ can lead to substantial effects on the proton decay and, depending on their sign, may enhance or even suppress the proton decay rate in gauge mediated channels.

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JHEP10(2006)030